

Multi-loop calculations in the MSSM

Matthias Steinhauser

KIT

June 2010, Loopfest IX, Stony Brook

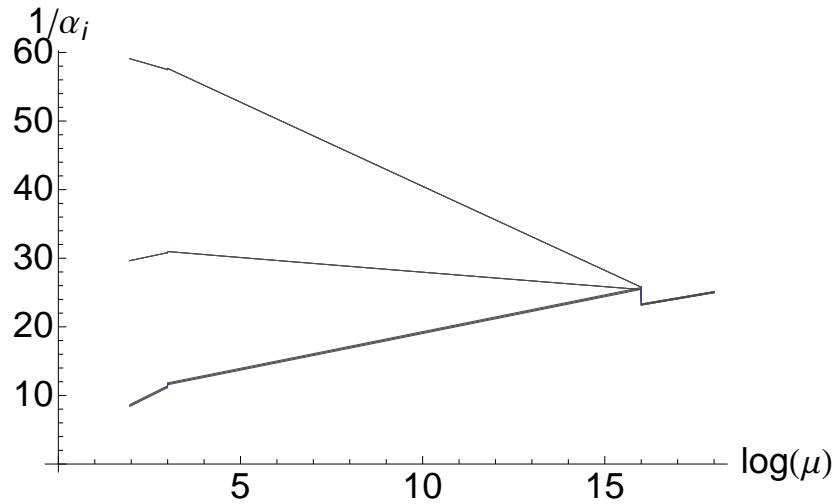
in collaboration with Robert Harlander, Philipp Kant, Luminita Mihaila



Outline

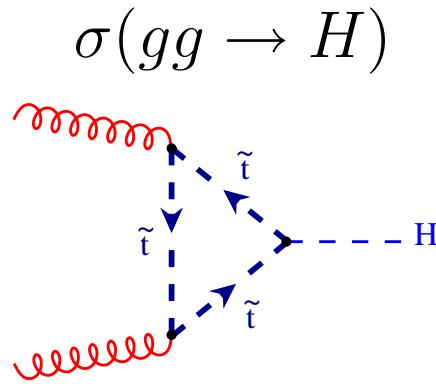
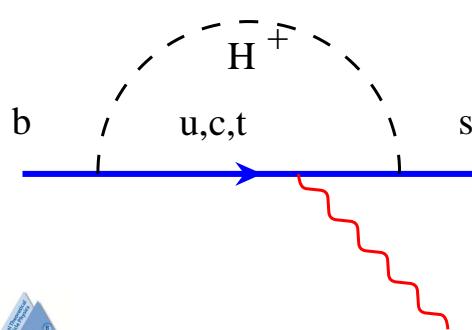
- Why many loops in the MSSM?
- SQCD β function to 3 loops
- M_h to 3 loops
- Conclusions

Motivation



M_h in the MSSM
is a prediction

rare SM processes
e.g.: $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$



β^{QCD}

$$\mu^2 \frac{d}{d\mu^2} \frac{\alpha_s}{\pi} = \beta = - \left(\frac{\alpha_s}{\pi} \right)^2 \left[\beta_0 + \frac{\alpha_s}{\pi} \beta_1 + \dots \right]$$

β_0 [Gross,Wilczek'73; Politzer'73]

β_1 [Caswell'74; Jones'74; Egorian,Tarasov'74]

β_2 [Tarasov,Vladimirov,Zharkov'80; Larin,Vermaseren'93]

β_3 [van Ritbergen,Vermaseren,Larin'97; Czakon'05]

β_2^{DRED} [Harlander,Jones,Kant,Mihaila,Steinhauser'06]

β_3^{DRED} [Harlander,Kant,Mihaila,Steinhauser'06]

β^{SQCD}

$$\mu^2 \frac{d}{d\mu^2} \frac{\alpha_s}{\pi} = \beta = - \left(\frac{\alpha_s}{\pi} \right)^2 \left[\beta_0 + \frac{\alpha_s}{\pi} \beta_1 + \dots \right]$$

β_0 [Cheng,Eichten,Li'74]

β_1 [Jones'75; Parkes,West'83; Jack,Osborn'83'85; Machacek,Vaughn'83'84'85; ...]

β_2 “NSVZ” [Jack,Jones,North'96]

“BFM” (gluon propagator) [Pickering,Gracey,Jones'01]

β_3 “NSVZ” [Jack,Jones,North'97] (up to a parameter)

Idea:

- check β_2 by explicit calculation
- use all WIs involving 3-point functions

3-loop beta function in the MSSM

- SQCD: $g, \tilde{g}, q, \tilde{q}, c, \epsilon$ [Harlander,Mihaila,Steinhauser'09]
- DRED [Siegel'79]
- all particles massless
- consider all vertices
compute corresponding 3- and 2-point functions
at 1, 2 and 3 loops

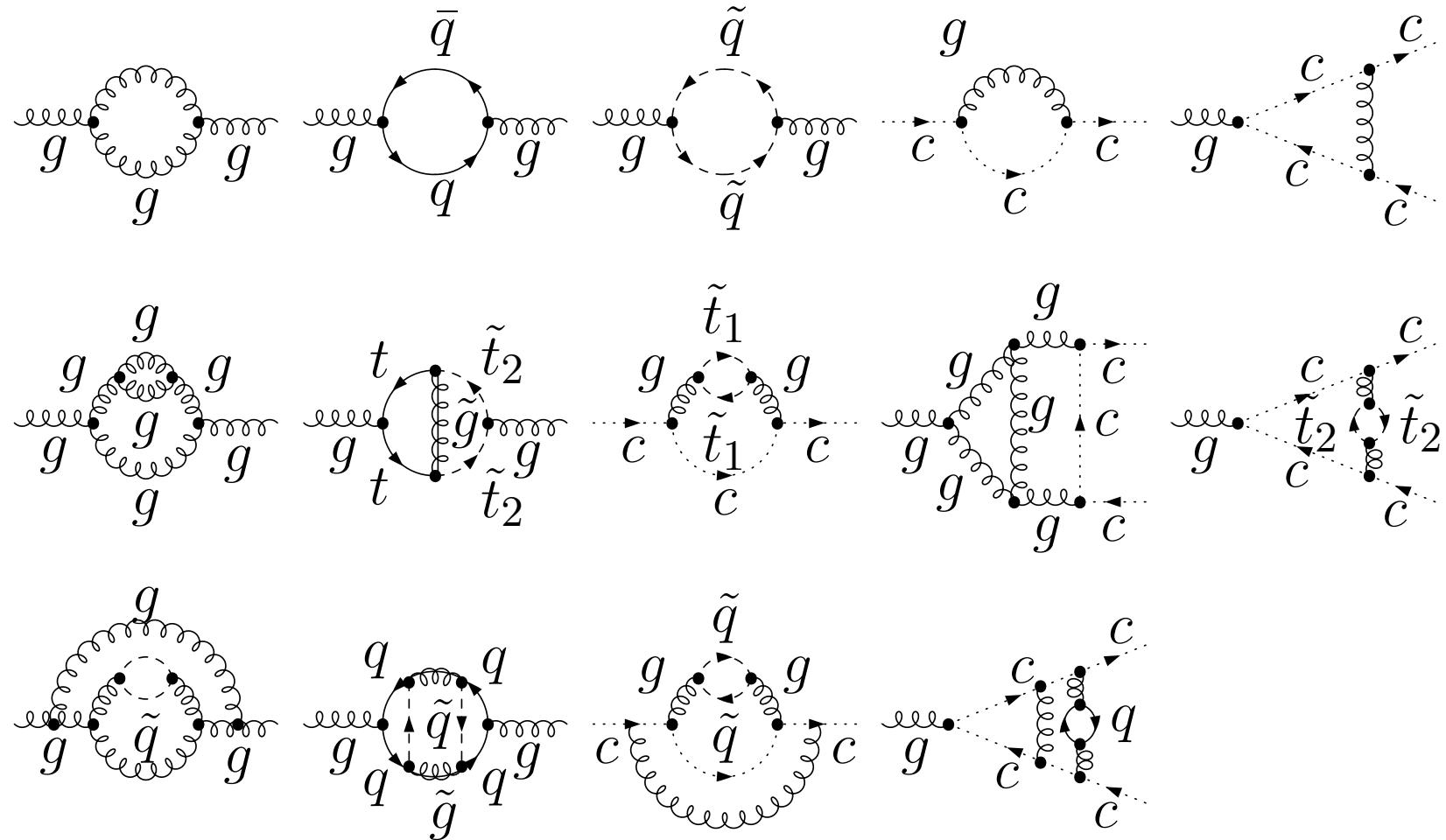
$$Z_g = \frac{Z_{\phi_1 \dots \phi_N}}{\sqrt{Z_{\phi_1} \dots Z_{\phi_N}}}$$

- diagrammatic calculation
- $\approx 300\,000$ Feynman diagrams

3-loop beta function in the MSSM

- diagrammatic calculation
- $\approx 300\,000$ Feynman diagrams

[Harlander,Mihaila,Steinhauser'09]



3-loop beta function in the MSSM

- diagrammatic calculation
- $\approx 300\,000$ Feynman diagrams

[Harlander,Mihaila,Steinhauser'09]

# loops	1	2	3
$c\bar{c}$	1	14	423
$q\bar{q}$	4	86	3583
$\epsilon\epsilon$	7	100	3902
$\tilde{g}\tilde{g}$	6	130	5577
$\tilde{q}\tilde{q}$	8	157	6760
gg	11	171	6954

# loops	0	1	2	3
$c\bar{c}g$	1	2	77	3920
$q\bar{q}\epsilon$	1	6	319	21669
$q\bar{q}g$	1	8	439	30078
$\tilde{g}\tilde{g}\epsilon$	1	8	445	31815
$q\tilde{q}\tilde{g}$	1	6	430	36868
$\epsilon\epsilon g$	1	14	618	38741
$\tilde{g}\tilde{g}g$	1	12	657	46314
$\tilde{q}\tilde{q}g$	1	12	674	52205
ggg	1	26	1105	70705

3-loop beta function in the MSSM

- diagrammatic calculation [Harlander,Mihaila,Steinhauser'09]
- $\approx 300\,000$ Feynman diagrams
- Dimensional Reduction applicable to 3 loops
- $$\beta(\alpha_s) = - \sum_{n \geq 0} \left(\frac{\alpha_s}{\pi} \right)^{n+2} \beta_n, [C_F = (n_c^2 - 1)/(2n_c), C_A = n_c, 2T_f = n_f]$$

$$\beta_0 = \frac{3}{4} C_A - \frac{1}{2} T_f, \quad \beta_1 = \frac{3}{8} C_A^2 - T_f \left(\frac{1}{2} C_F + \frac{1}{4} C_A \right),$$

$$\beta_2 = \frac{21}{64} C_A^3 + T_f \left(\frac{1}{4} C_F^2 - \frac{13}{16} C_A C_F - \frac{5}{16} C_A^2 \right) + T_f^2 \left(\frac{3}{8} C_F + \frac{1}{16} C_A \right)$$

- 3-loop quark mass anomalous dimension

$$\gamma_q(\alpha_s) = - \sum_{n \geq 0} \left(\frac{\alpha_s}{\pi} \right)^{n+1} \gamma_n^q$$

- 3-loop gluino mass anomalous dimension

$$\gamma_{\tilde{g}}(\alpha_s) = - \sum_{n \geq 0} \left(\frac{\alpha_s}{\pi} \right)^{n+1} \gamma_n^{\tilde{g}}$$

$\gamma_n^{\tilde{g}} = (n + 1)\beta_n$ verified to 3 loops

Higgs boson mass in the MSSM

- MSSM: 5 Higgs bosons: h, H, A, H^\pm
- prediction of M_h
- tree-level:

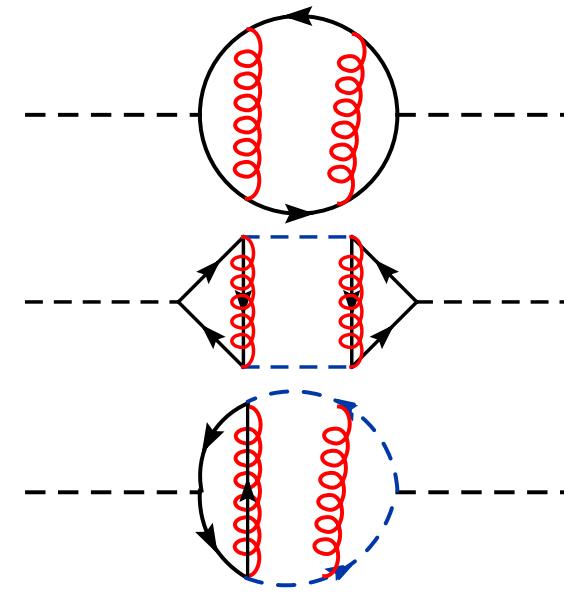
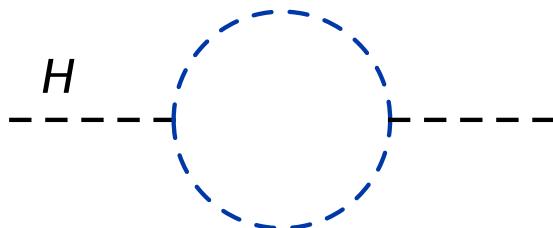
$$\mathcal{M}_{H,\text{tree}}^2 = \frac{\sin 2\beta}{2} \begin{pmatrix} M_Z^2 \cot \beta + M_A^2 \tan \beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2 \tan \beta + M_A^2 \cot \beta \end{pmatrix}$$

$$\Leftrightarrow M_h \leq M_Z$$

- quantum corrections:

$$\mathcal{M}_H^2 = \mathcal{M}_{H,\text{tree}}^2 + \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1 \phi_2} \\ \hat{\Sigma}_{\phi_1 \phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}$$

squark



Known results

- $\Delta M_h^{(1)}$: [Ellis,Ridolfi,Zwirner'91; Okada,Yamaguchi,Yanagida'91; Haber,Hempfling'91]
 - $\Delta M_h^{(2)}$: [... many authors ..., Haber et al.'97; Degrassi,Slavich,Zwirner'01; Allanach et al.'04; Heinemeyer'06; Frank et al.'06; Martin'03]
 - $\Delta M_h^{(3)}$, leading logs: [Martin'07]
-
- FeynHiggs [Frank,Hahn,Heinemeyer,Hollik,Rzehak,Weiglein]
 - CPSuperH [Lee,Pilaftsis,Carena,Choi,Drees,Ellis,Wagner]
-
- remaining uncertainty: 3-5 GeV [Degrassi et al.'02; Allanach et al.'04]
 - LHC: $\delta M_h \sim 100 - 200$ MeV
 - ILC: $\delta M_h \sim 50$ MeV
-
- $\Delta M_h^{(3)}$: [Kant,Harlander,Mihaila,Steinhauser'08'10]

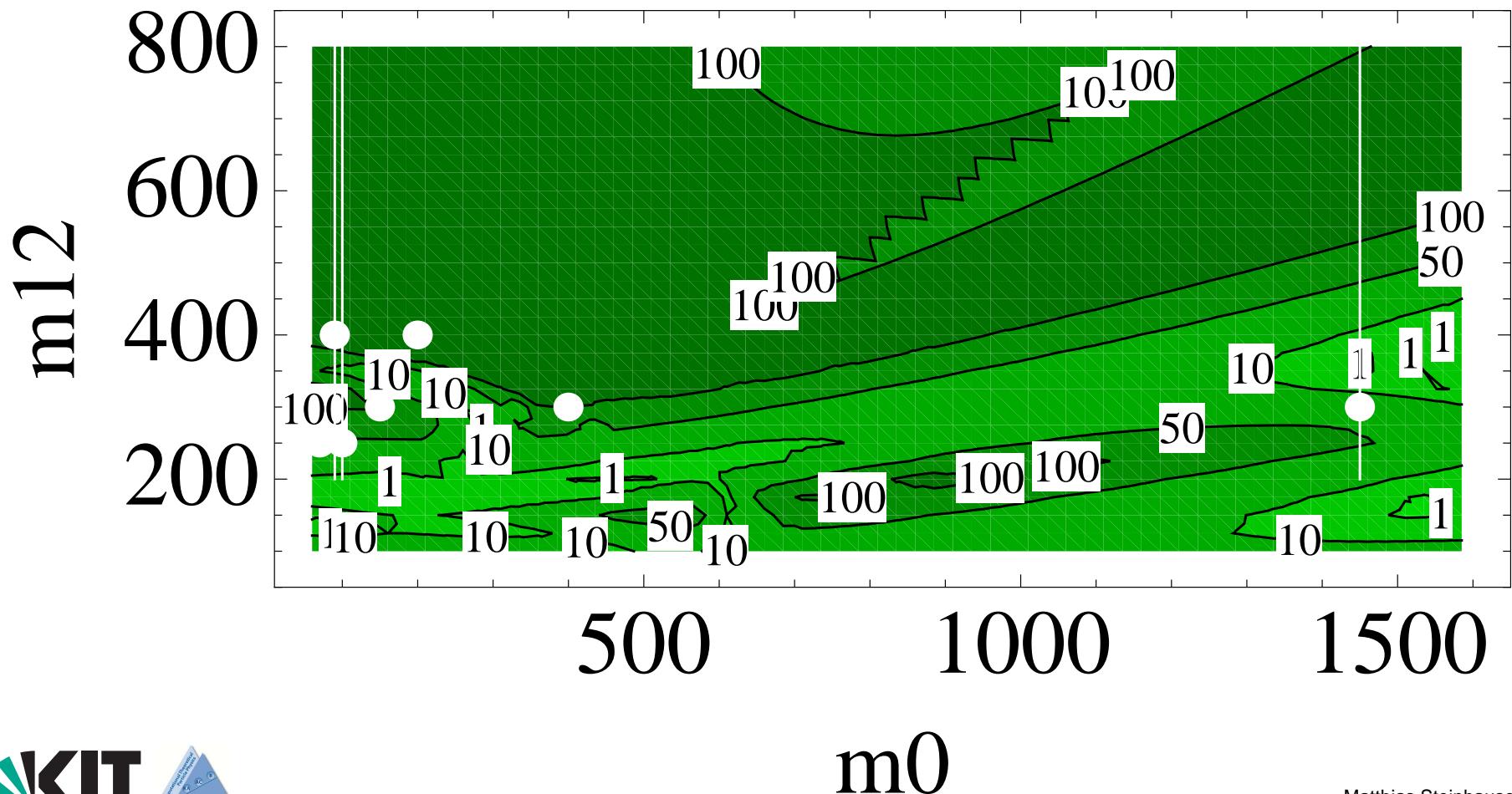
Framework

- $g, \tilde{g}, t, \tilde{t}, q, \tilde{q}, c, \epsilon$
- DRED [Siegel'79]
- $\hat{\Sigma}(q = 0)$, leading contribution: $\sim G_F m_t^4 \alpha_s^2$
- $\overline{\text{DR}}$ renormalization
- mass scales: $m_t, m_{\tilde{t}1}, m_{\tilde{t}2}, m_{\tilde{g}}, m_{\tilde{q}}$
- consider hierarchies + asymptotic expansion
 - (h3) $m_{\tilde{q}} \approx m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}}$
 - (h4) $m_{\tilde{q}} \gg m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}}$
 - (h5) $m_{\tilde{q}} \gg m_{\tilde{t}_2} \gg m_{\tilde{t}_1} \approx m_{\tilde{g}}$
 - (h6) $m_{\tilde{q}} \gg m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_{\tilde{t}_1}$
 - (h6b) $m_{\tilde{q}} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_{\tilde{t}_1}$
 - (h9) $m_{\tilde{q}} \approx m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \gg m_{\tilde{g}}$
- $\sim 30\,000$ diagrams and up to $\sim 100\,000$ subdiagrams

2 loops: compare to exact result

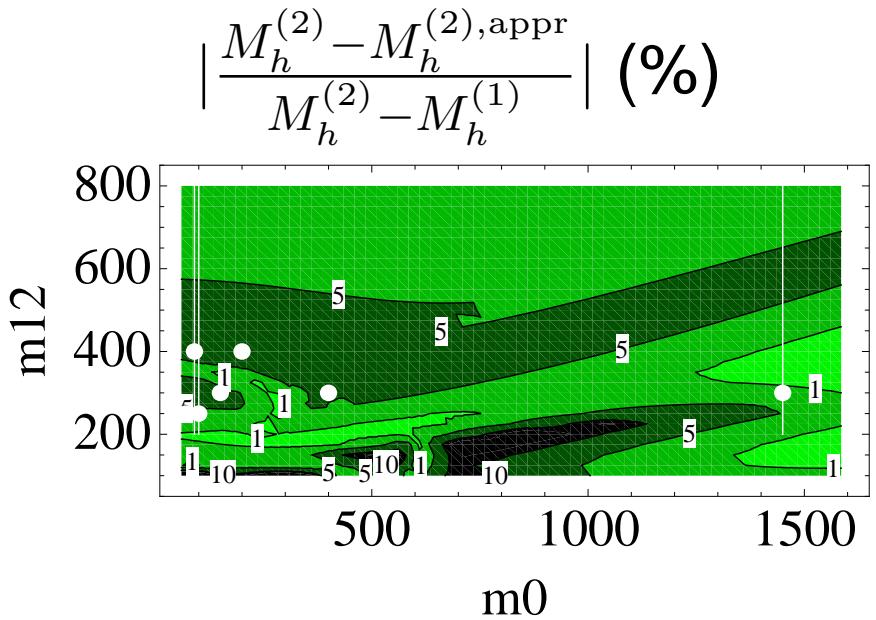
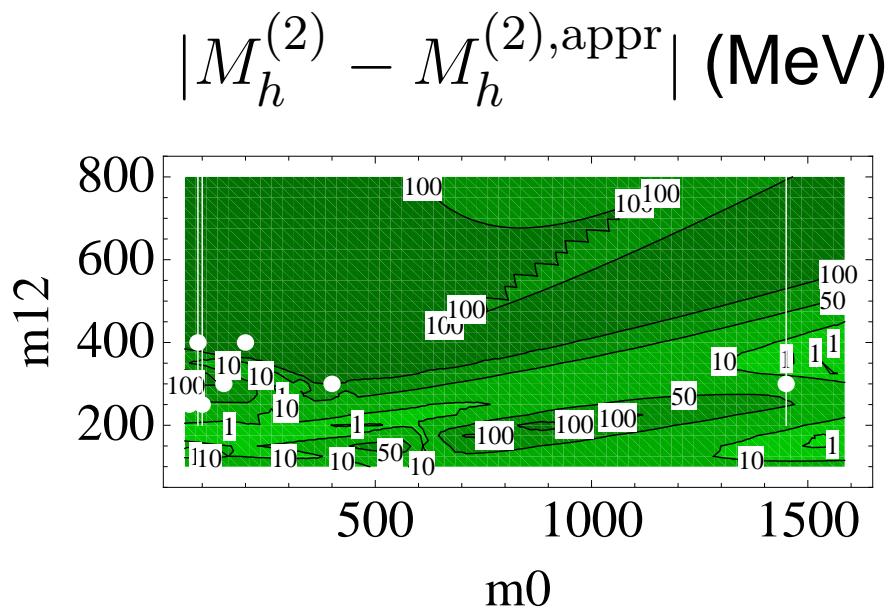
msugra: $\tan \beta = 10$, $A_0 = 0$, $\mu_{\text{SUSY}} > 0$,
 $60 \text{ GeV} < m_0 < 1600 \text{ GeV}$, $100 \text{ GeV} < m_{1/2} < 800 \text{ GeV}$

$$|M_h^{(2)} - M_h^{(2),\text{appr}}| \text{ (MeV)}$$

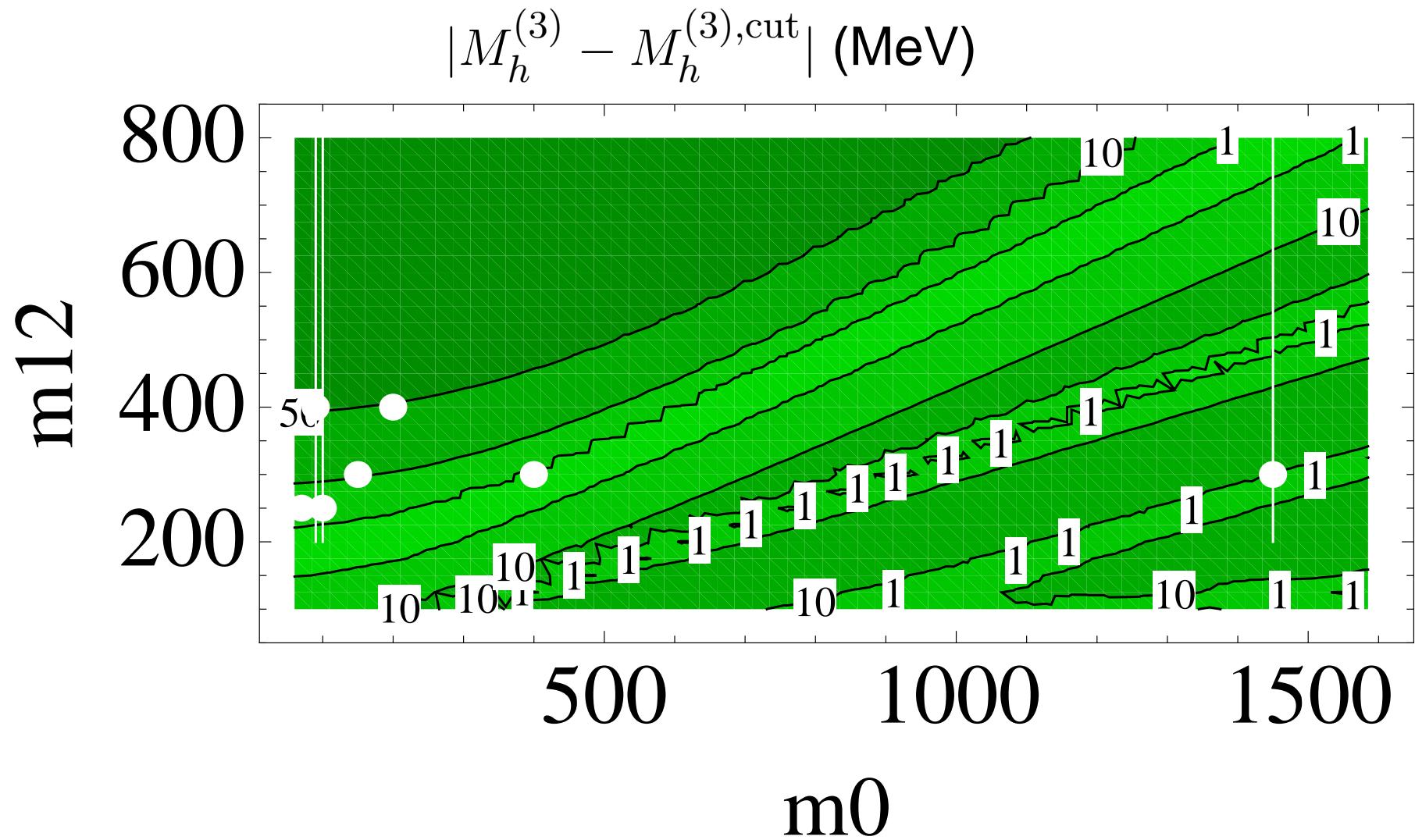


2 loops: compare to exact result

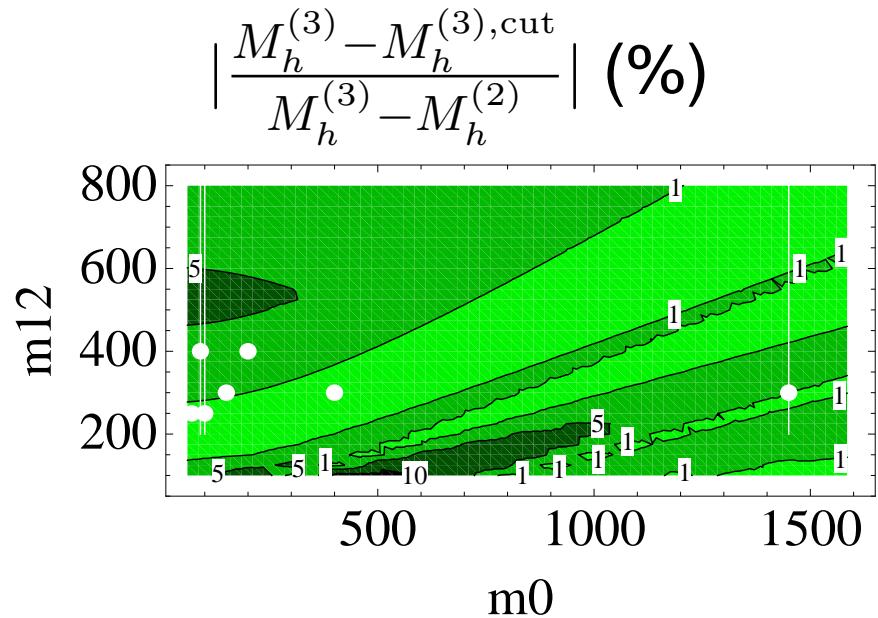
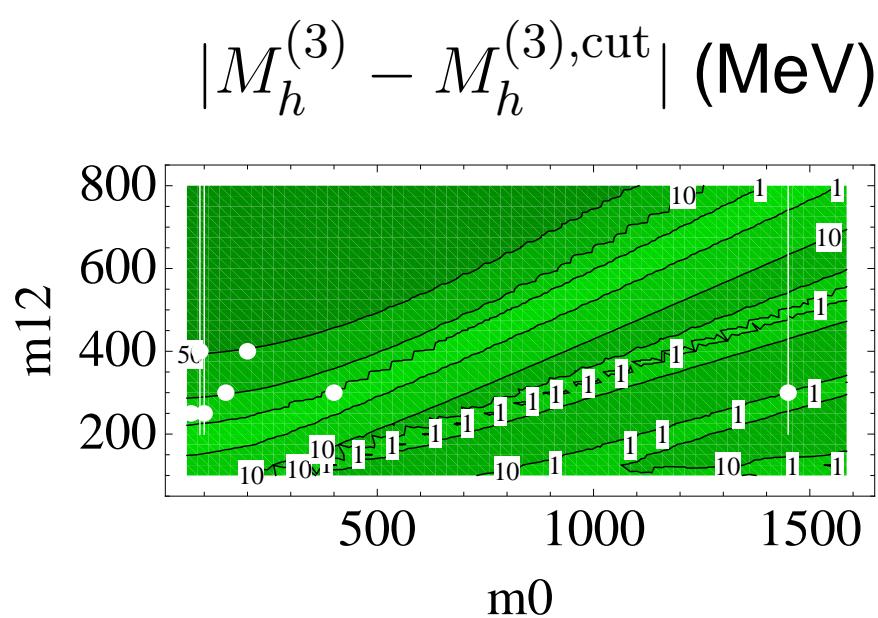
msugra: $\tan \beta = 10$, $A_0 = 0$, $\mu_{\text{SUSY}} > 0$,
 $60 \text{ GeV} < m_0 < 1600 \text{ GeV}$, $100 \text{ GeV} < m_{1/2} < 800 \text{ GeV}$



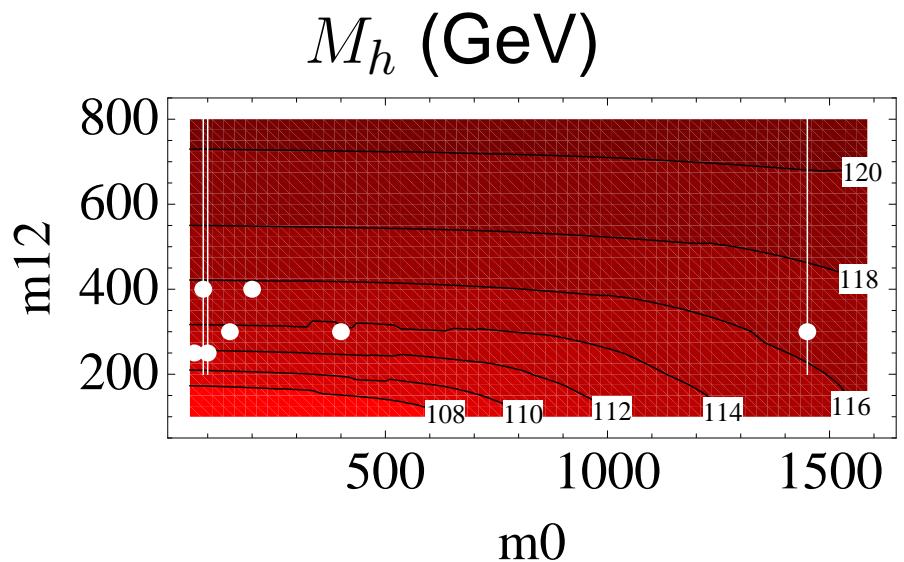
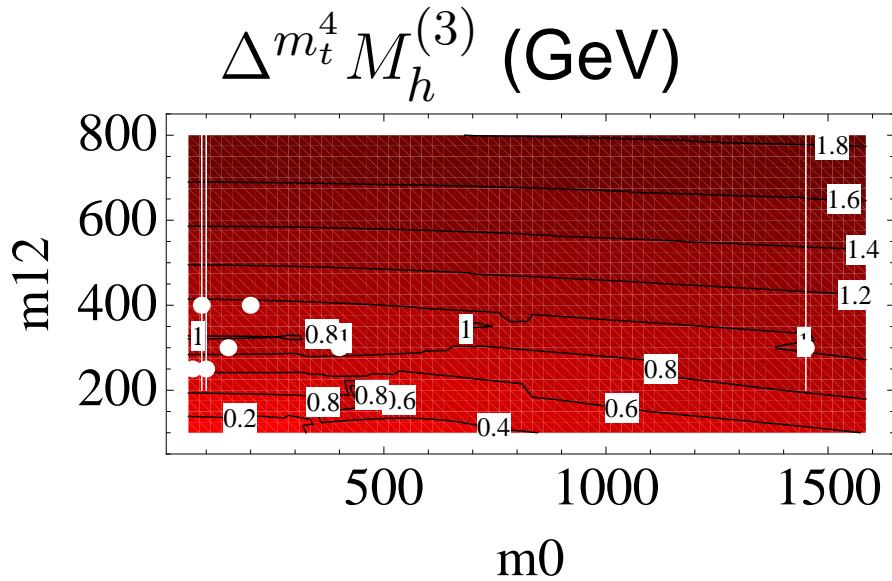
3 loops: check expansion



3 loops: check expansion



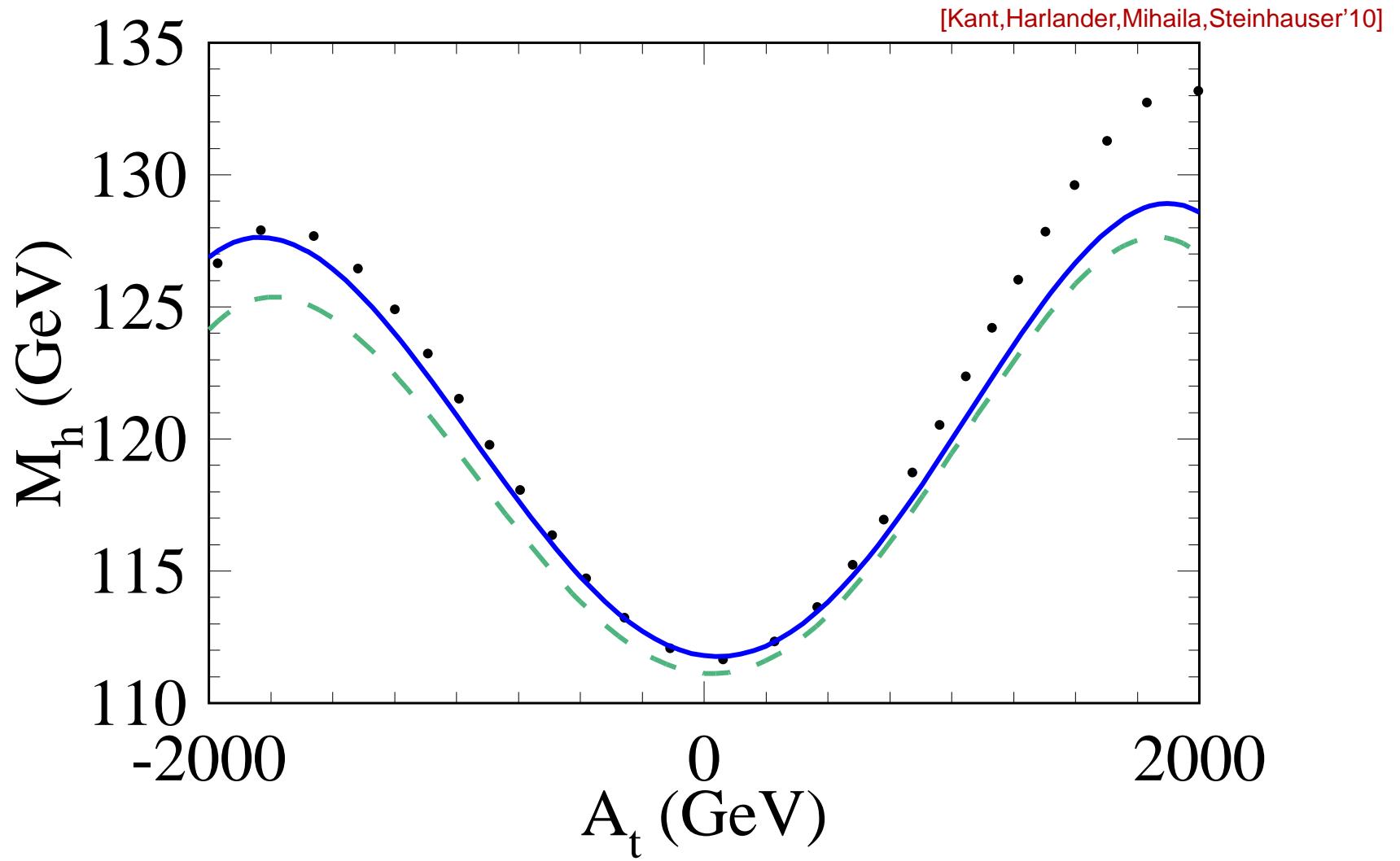
3 loops: results



Uncertainties:

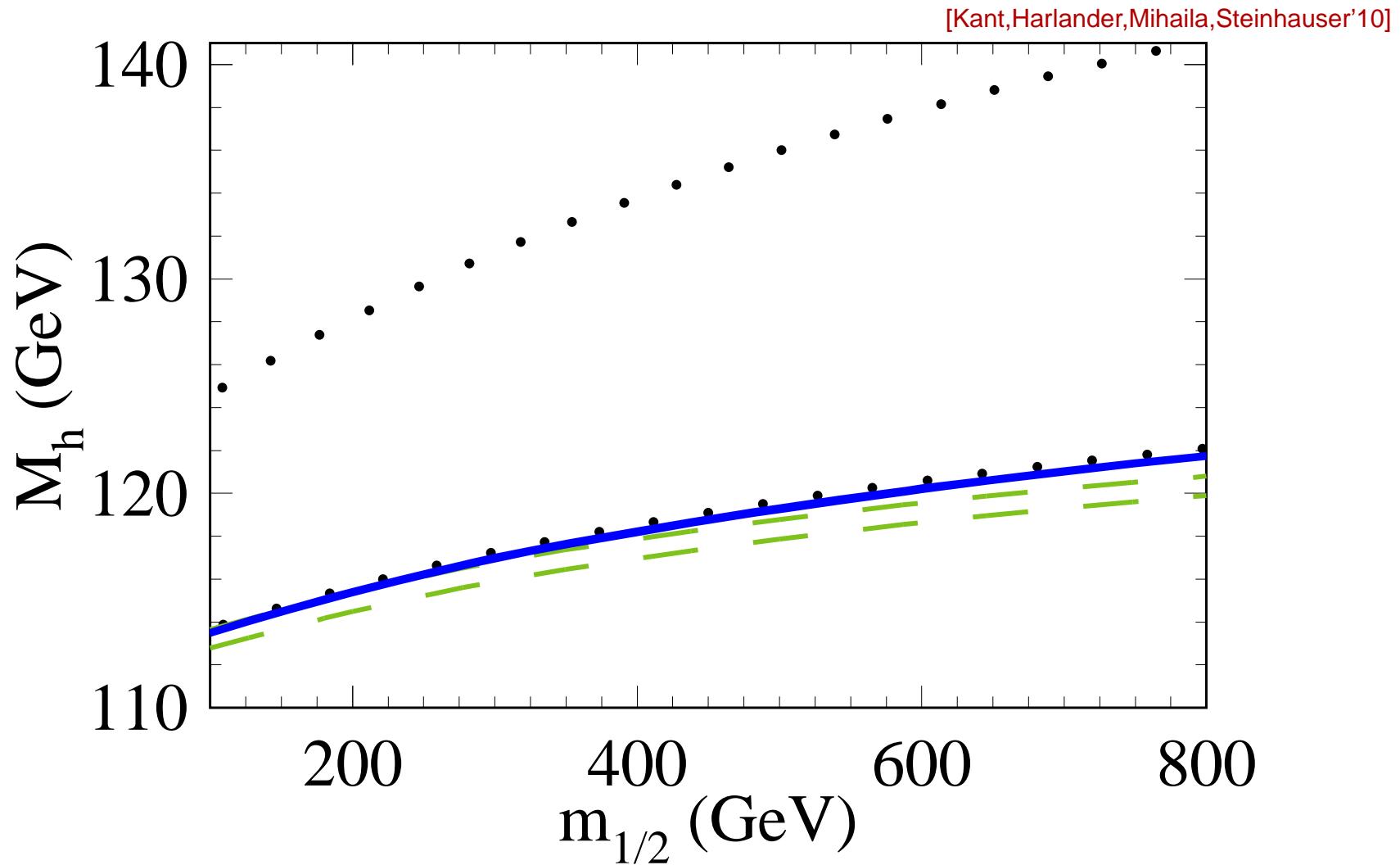
theory	parametric	
$\Delta^{\text{rem}} M_h^{(3)}$:	100 MeV	$\delta M_t = 1.3 \text{ GeV}$ $\leq 350 \dots 1000 \text{ MeV}$
approx. for $\Delta^{m_t^4} M_h^{(3)}$:	100 MeV	$(m_{1/2} = 100 \dots 1000 \text{ GeV})$
h.o. ($\Delta M_h^{(3)} \approx -\frac{1}{2} \Delta M_h^{(2)}$):	100 ... 1000 MeV	$\delta \alpha_s = 0.0020 \text{ GeV}$ $\leq 80 \dots 600 \text{ MeV}$
$q^2 \neq 0, \text{ew}, \dots$	200 MeV	
$\delta(\text{theory}) \approx \delta(\text{param.})$		

M_h to 3 loops

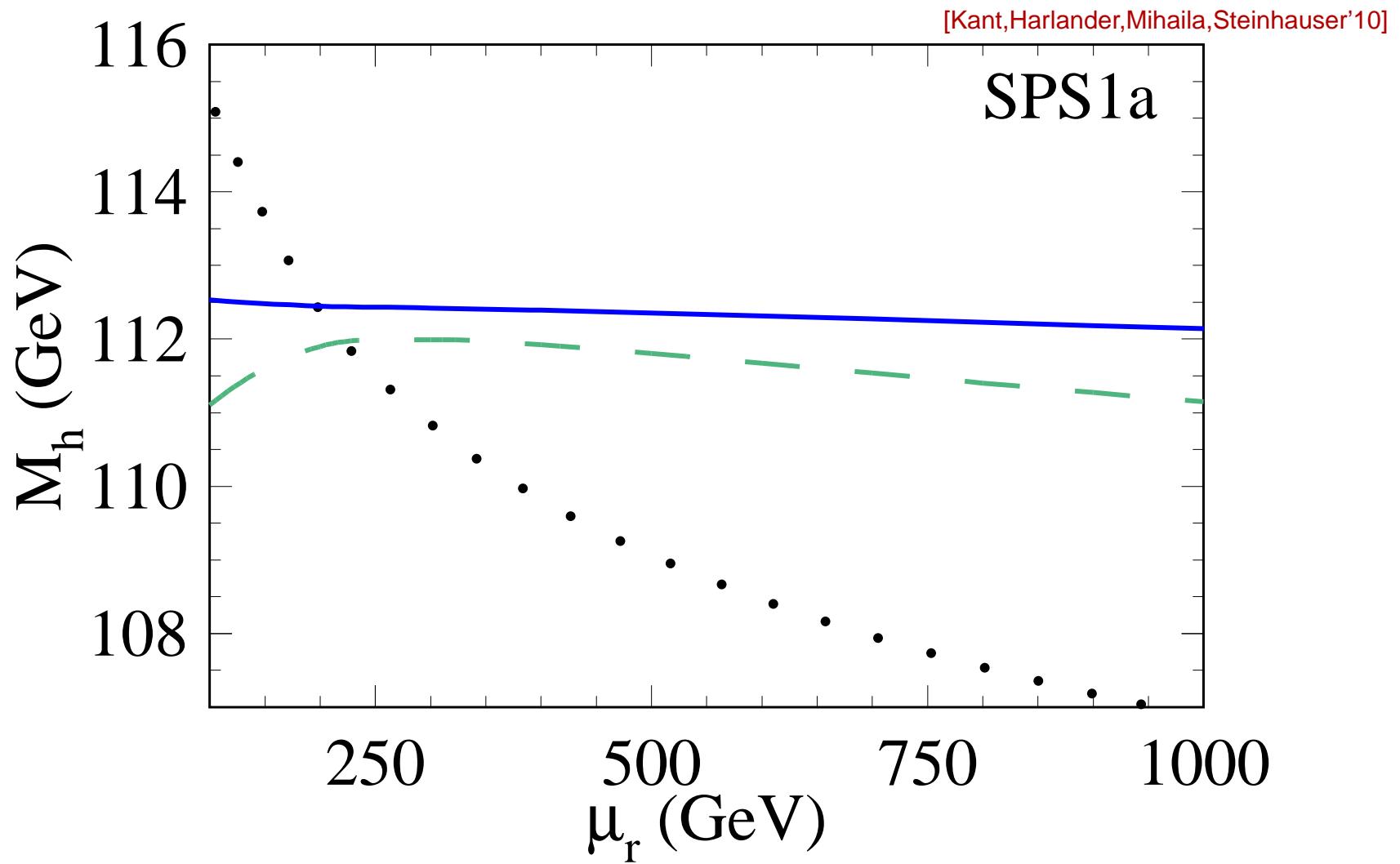


3-loop corrections \approx few GeV \gg 100 MeV \approx exp. uncertainty

Comparison: OS – $\overline{\text{DR}}$

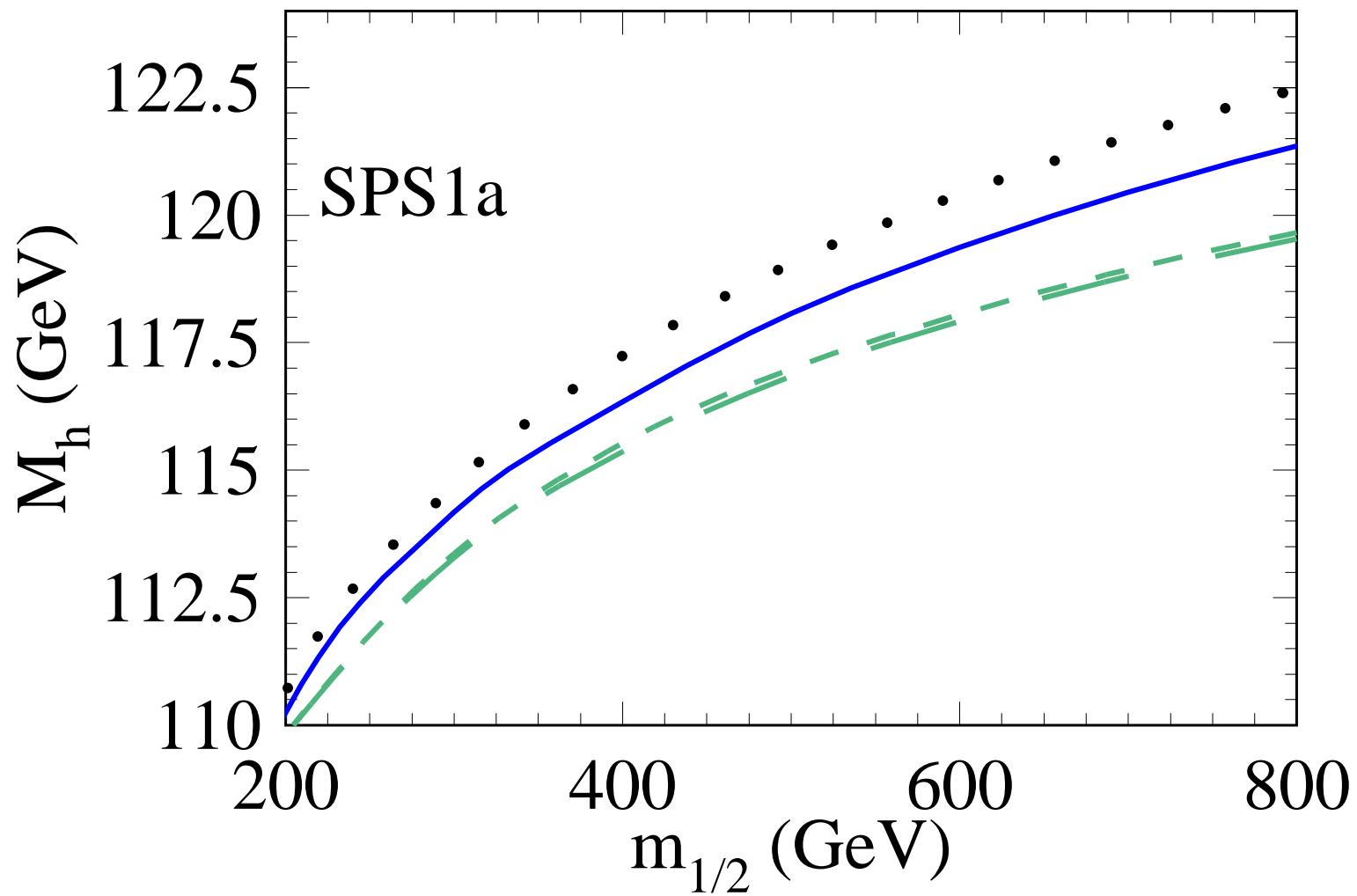


Renormalization scale dependence



SPS1a

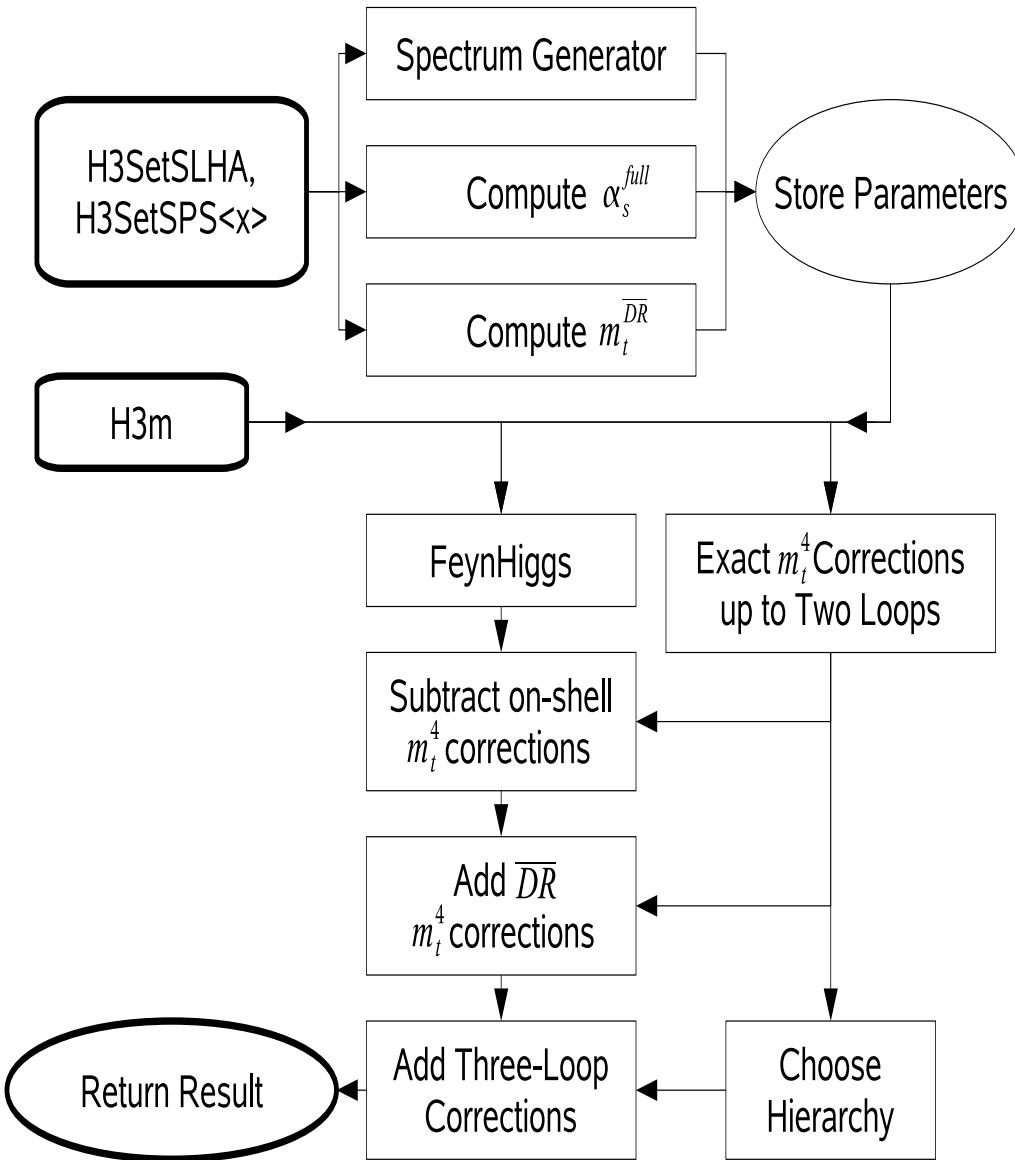
[Kant,Harlander,Mihaila,Steinhauser'10]



SPS at 2 and 3 loops

	$M_h^{(2)}$ (GeV)	$M_h^{(2),\text{appr}}$ (GeV)	$M_h^{(3)}$ (GeV)	$M_h^{(3),\text{cut}}$ (GeV)	optimal hierarchy
SPS1a	111.81	111.84	112.46	112.45	h6b
SPS1a'	113.26	113.27	113.92	113.92	h6b
SPS1b	115.53	115.64	116.49	116.44	h3
SPS2	115.65	115.77	116.67	116.61	h5
SPS3	114.63	114.77	115.59	115.52	h3
SPS4	113.73	113.77	114.82	114.81	h6
SPS5	111.66	111.83	112.02	111.92	h3
SPS7	112.20	112.21	113.04	113.04	h3
SPS8	114.19	114.20	115.03	115.02	h3

H3m



[exact 2-loop m_t^4 expression from [Degrassi,Slavich,Zwirner'01]; OS- \overline{DR} for M_t from [Martin'05]]

Mathematica 7.0 for Linux x86 (64-bit)
Copyright 1988-2008 Wolfram Research, Inc.

In[1]:= Needs["H3`"];

RunDec: a Mathematica package for running and decoupling
strong coupling and quark masses
by K.G. Chetyrkin, J.H. Kuhn and M. Steinhauser (January

In[2]:= H3SetSPS1a[300.];

H3GetSLHA::TSIL: Using TSIL by S.P. Martin.

FeynHiggs 2.6.5

built on Dec 20, 2008

T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weigle
<http://www.feynhiggs.de>

FHHiggsCorr contains code by:

P. Slavich et al. (2-loop rmSSM Higgs self-energies)

Loading Results for hierarchy h3

Loading Results for hierarchy h3

Loading Results for hierarchy h6b2qg2

Loading Results for hierarchy h6b2qg2

In[3]:= H3m[]

Loading Results for hierarchy h6b2qg2

Out[3]= {mh -> 114.176}

Summary

- 3-loop calculations in the MSSM
- β function in SUSY QCD
- Higgs boson mass to 3 loops
- $\Delta M_h^{(3\text{loops})} > \Delta M_h^{(\text{LHC,ILC})}$
- H3m: most precise value for M_h
- $\delta M_h < 1 \text{ GeV}$